Introduction to Applied Math

Homework 1

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Complete the following exercises. Show **all** your work and justify your answers. **Unjustified answer will not be evaluated.** You may use any method you have learned in class.

Basic Exercises

1. Verify by differentiation that the function $y(x) = Ce^{-2x} + \frac{1}{2}x - \frac{1}{4}$ is a general solution to the differential equation:

$$\frac{dy}{dx} + 2y = x\,.$$

2. Find the general solution to the following separable differential equation. Express y explicitly as a function of x if possible:

$$\frac{dy}{dx} = \frac{x^2}{y^2}\,.$$

3. Solve the following first-order linear initial value problem using an integrating factor:

$$x\frac{dy}{dx} + 2y = 4x^2\,, \quad y(1) = 2\,.$$

4. Determine whether the following differential form is exact. If it is exact, find the general potential function $F(x, y)$ such that $\partial_x F = M(x, y)$, $\partial_y F = N(x, y)$:

$$M(x, y)dx + N(x, y)dy = (2xy^2 + 1)dx + (2x^2y)dy\,.$$

5. Solve the following homogeneous first-order differential equation:

$$\frac{dy}{dx} = \frac{y^2 + x^2}{xy}\,.$$

6. A population of bacteria $P(t)$ grows according to the logistic equation:

$$\frac{dP}{dt} = 0.05P\left(1 - \frac{P}{1000}\right)\,.$$

Find the equilibrium solutions for this population and state whether each equilibrium is stable or unstable.

Challenge Exercises

You must complete at least one of the following challenge exercises. You may attempt more than one if you wish.

1. **(Euler's Method)** Consider the initial value problem:

$$\frac{dy}{dx} = x + y\,, \quad y(0) = 1\,.$$

   Use Euler's method with a step size of $h = 0.5$ to approximate the value of the solution at $x = 1$. (You will need to perform two steps by hand).

2. **(Bernoulli Equations)** A Bernoulli differential equation has the form:

$$\frac{dy}{dx} + P(x)y = Q(x)y^n\,,$$

   where $n \neq 0$ and $n \neq 1$. Prove that the substitution $v = y^{1-n}$ transforms this nonlinear equation into a linear first-order equation in terms of v and x.