



**Exercise 2**

Compute the limits. If one does not exist, show why using two paths.

(a)  $\lim_{z \rightarrow 2i} \frac{z^2 + 4}{z - 2i}$

(b)  $\lim_{z \rightarrow 0} \frac{\operatorname{Im}(z)}{z}$

**Exercise 3**

Let  $z = 1 - i$ .

(a) Write  $z$  in exponential form.

(b) Compute  $z^{10}$  and express the answer in the form  $r(\cos(\theta) + i \sin(\theta))$ .

**Exercise 4**

Consider  $u(x, y) = x^2 - y^2 - 2y$ .

(a) Verify that  $u(x, y)$  is harmonic.

(b) Find the harmonic conjugate  $v(x, y)$  such that  $f = u + iv$  is analytic.

(c) **Bonus:** Write  $f(z)$  explicitly in terms of  $z$ .

**Exercise 5 - Graduate Only**

**Proposition:** *If  $v$  is the harmonic conjugate of  $u$ , then  $-u$  is the harmonic conjugate of  $v$ .*

- (a) Verify the previous statement.