

Exercise 2

Compute the limits. If one does not exist, show why using two paths.

(a) $\lim_{z \rightarrow 1+i} \frac{z^2 - 2z + 2}{z - (1+i)}$

(b) $\lim_{z \rightarrow 0} \left(\frac{z}{\bar{z}} \right)^2$

Exercise 3

Let $z = -\sqrt{3} + i$.

(a) Write z in exponential form.

(b) Compute z^{12} and express the answer in the form $x + iy$.

Exercise 4

Consider $u(x, y) = x^3 - 3xy^2$.

(a) Verify that $u(x, y)$ is harmonic.

(b) Find the harmonic conjugate $v(x, y)$ such that $f = u + iv$ is analytic.

(c) **Bonus:** Write $f(z)$ explicitly in terms of z .

Exercise 5 - Graduate Only

Proposition: *If $f(z)$ is analytic in a domain D and its modulus $|f(z)|$ is constant throughout D , then $f(z)$ must be constant in D .*

- (a) Prove the proposition above. (*Hint: Start with $|f(z)|^2 = u^2 + v^2 = c^2$ and differentiate with respect to x and y . Then use the Cauchy-Riemann equations.*)