Complex Analysis Worksheet

Chapter 1: Complex Numbers

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1. **Basic Algebra.**
   Let $z_1 = 3 - 2i$ and $z_2 = 1 + 4i$. Compute the following and express the result in the standard form $x + iy$:

   (a) $z_1 \cdot \overline{z_2}$

   (b) $\dfrac{z_1}{z_2}$

   (c) $|z_1 - 2z_2|^2$

2. **Exponential Form and Arguments.**
   Consider the complex number $z = -2\sqrt{3} + 2i$.

   (a) Calculate the modulus $|z|$.

   (b) Find the principal argument, $\mathrm{Arg}(z)$.

   (c) Write z in exponential form $re^{i\theta}$.

3. **Powers of Complex Numbers.**
   Use the exponential forms to compute the exact value of:

$$(1 - i)^{12}$$

4. **Powers of Complex Numbers.**
   Use De Moivre's Formula to compute the exact value of:

$$\left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)^4$$

5. Roots of Complex Numbers.
Find all distinct cube roots of $8i$.

(a) Write the roots in exponential form.

(b) Sketch the position of these roots on the complex plane.

6. Regions in the Plane.
Sketch the set of points z in the complex plane that satisfy the following inequality:

$$1 < |z - (1 + i)| \leq 3$$

Determine if this set is open, closed, or neither.

7. The Parallelogram Law.
Prove that for any two complex numbers z_1 and z_2, the following identity holds:

$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$$

Hint: Use the property that $|w|^2 = w\bar{w}$ *and expand the terms.*