Complex Analysis Worksheet

Chapter 4: Integrals

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1. Complex-Valued Functions of a Real Variable.
Evaluate the following definite integrals by treating the complex parts separately or by using formal antiderivatives:

(a) $\displaystyle\int_0^{\pi/4} e^{(1+i)t}\,dt$

(b) $\displaystyle\int_0^{1} (1+it)^3\,dt$

2. Parameterization of Contours.
Give a smooth parametric representation $z(t) = x(t) + iy(t)$ for each of the following contours. Be sure to specify the interval $[a, b]$ for the parameter t.

(a) The straight line segment directed from $z_1 = -1 + i$ to $z_2 = 3 + 2i$.

(b) The upper half of the circle $|z - 2| = 3$, traversed counterclockwise (i.e., from $z = 5$ to $z = -1$).

3. Basic Contour Integration.
Evaluate the integral

$$\int_C \bar{z}\,dz$$

where C is the right half of the circle $|z| = 2$, traversed from $z = -2i$ to $z = 2i$.

4. The Integral of $1/z$ and Radius Independence.
Let C_R denote the circle $|z| = R$ (where $R > 0$), traversed once in the counterclockwise direction.

(a) Parameterize the contour C_R and use it to evaluate the contour integral:

$$\oint_{C_R} \frac{1}{z}\,dz$$

(b) Based on your calculation, what happens to the value of the integral as R grows larger or smaller? Conclude that the integral of $1/z$ around the origin is strictly independent of the radius R.

5. **Integrals of Integer Powers.**
   Let C_R be the same counterclockwise circle $|z| = R$ as in the previous exercise. Evaluate the integral:
$$\oint_{C_R} z^n \, dz$$
   where n is any integer such that $n \neq -1$.
   Hint: Parameterize z in the same way and apply Euler's formula to the result.

6. **Integration over a Polygonal Path.**
   Evaluate the integral
$$\int_C \mathrm{Re}(z) \, dz$$
   where C is the polygonal path consisting of the line segment from 0 to $1+i$, followed by the line segment from $1+i$ to $2i$.

7. **Principal Branch Integration.**
   Evaluate the integral
$$\int_C \mathrm{Log}(z) \, dz$$
   where C is the upper half of the unit circle $|z| = 1$, traversed from $z = 1$ to $z = -1$.
   Hint: Use the parameterization $z(\theta) = e^{i\theta}$ for $0 \leq \theta \leq \pi$ and remember the definition of the principal logarithm.

8. **Green's Theorem Application.**
   let D be a closed, simply conneted domain in the complex plane with a positively oriented, piecewise smooth boundary C. Let $f(z) = u(x, y) + iv(x, y)$ be an analytic function with continuous derivative on an open set containing D and its boundary C. Use Green's theorem to show that:
$$\int_C f(z) \, dz = 0$$
   where $z = x + iy$ and u and v are the real and imaginary parts of $f(z)$, respectively.
   Where did you use the analyticity and the continuity of the first derivative?