Complex Analysis Worksheet

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1. Independence of Path (Antiderivatives).
Evaluate the following integrals using the antiderivative. Specify the principal branch if you use a multiple-valued function.

(a) $\displaystyle\int_{i}^{i/2} e^{\pi z}\,dz$

(b) $\displaystyle\int_{0}^{\pi+2i} \cos\left(\frac{z}{2}\right)\,dz$

(c) $\displaystyle\int_{C} \frac{1}{z}\,dz$, where C is an arc from $z = 1$ to $z = -i$ restricted to the right half-plane.

2. The Cauchy-Goursat Theorem.
Let C be the unit circle $|z| = 1$, traversed counterclockwise. Evaluate the following integrals, and explicitly state why the Cauchy-Goursat theorem applies (or does not apply).

(a) $\displaystyle\int_{C} z^3 e^{z^2}\,dz$

(b) $\displaystyle\int_{C} \frac{z^2+4}{z-3}\,dz$

(c) $\displaystyle\int_{C} \frac{1}{z^2+4}\,dz$

3. Cauchy Integral Formula.
Use the Cauchy Integral Formula to evaluate the following integrals. In each case, C is a simple closed contour oriented counterclockwise.

(a) $\displaystyle\int_{|z|=2} \frac{e^z}{z-1}\,dz$

(b) $\displaystyle\int_{|z-i|=1} \frac{\sin(\pi z)}{z-i}\,dz$

4. Isolating the Analytic Part.

Evaluate the integral

$$\int_C \frac{z}{(z^2+9)(z+i)}\,dz$$

where C is the circle $|z+i|=1$ traversed counterclockwise.

Hint: Rewrite the integrand in the form $\frac{f(z)}{z-z_0}$ *where* $f(z)$ *is analytic inside and on* C.

5. Cauchy Integral Formula for Derivatives.

Recall the generalized formula: $\displaystyle\int_C \frac{f(z)}{(z-z_0)^{n+1}}\,dz = \frac{2\pi i}{n!} f^{(n)}(z_0)$. Use it to evaluate the following integrals over the counterclockwise circle $C: |z|=2$.

(a) $\displaystyle\int_C \frac{e^{2z}}{z^3}\,dz$

(b) $\displaystyle\int_C \frac{z^4+2z+1}{(z-i)^2}\,dz$